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## ABSTRACT

An analogy between inelastic strains caused by temperature and those caused by creep is presented in terms of isotropic elasticity. It is shown how the theoretical aspects can be blended with existing finite-element computer programs to exact a piecewise linear solution. The creep effect is determined by using the thermal stress computational approach, if appropriate alterations are made to the thermal expansion of the individual elements. The overall transient solution is achieved by consecutive piecewise linear iterations. The total residue caused by creep is obtained by accumulating creep residues for each iteration and then resubmitting the total residues for each element as an equivalent input. A typical creep law is tested for incremental time convergence. The results indicate that the approach is practical, with a valid indication of the extent of creep after approximately 20 hr of incremental time. The general analogy between body forces and inelastic strain gradients is discussed with respect to how an inelastic problem can be worked as an elastic problem.

## INTRODUCTION

Residual stresses caused by creep are important to airframe designers because the stresses manifest themselves as excessive deformations, local buckling, or structural failure (refs. 1 to 4). Effective airframe design should include the complete utilization of available analysis tools. The purpose of this paper is to present an analogous relationship for predicting residual stresses caused by creep in built-up structures. Using existing finite-element computer programs, it will be shown how an analogy between inelastic strains representing temperature and creep presents a viable approach to a complex problem. The mathematics of the analogy are presented in terms of isotropic elasticity, and practical applications are also considered.

## NOMENCLATURE

E	Young's modulus, Pa (lb/in <sup>2</sup> )
G	shear modulus, Pa (lb/in <sup>2</sup> )
T	temperature, K (°F)
t	time, hr
$\alpha$	coefficient of thermal expansion, m/m K (in/in °F)
$\delta_{ij}$	Kronecker delta
$\epsilon$	strain, m/m (in/in)
$\eta$	arbitrary constant
$\nu$	Poisson's ratio

$\sigma$  stress, Pa (lb/in<sup>2</sup>)

Subscripts:

AVG average

C creep

i,j,k,l,m integers

P plasticity

T temperature

## RESULTS AND DISCUSSION

### Strain Relations

Strain can be considered to be composed of an elastic part and an inelastic part, and can be represented as

$$\epsilon_{ij} = \underbrace{\epsilon'_{ij}}_{\text{elastic}} + \underbrace{\epsilon''_{ij}}_{\text{inelastic}} \quad (1)$$

Inelastic strains,  $\epsilon''_{ij}$ , may be composed of several parts, such as temperature, creep, and plasticity, in which case

$$\epsilon''_{ij} = \underbrace{\epsilon''_{ijT}}_{\text{temperature}} + \underbrace{\epsilon''_{ijC}}_{\text{creep}} + \underbrace{\epsilon''_{ijP}}_{\text{plasticity}} \quad (2)$$

Elastic strains (that is those that cause stress) may be written as the difference between the total strain and the inelastic strain:

$$\epsilon'_{ij} = \epsilon_{ij} - \epsilon''_{ij} \quad (3)$$

The elastic strain,  $\epsilon'_{ij}$ , is related to stress through the generalized Hooke's law (ref. 5) for isotropic elasticity

$$\epsilon'_{ij} = \frac{\sigma_{ij}}{2G} - \left( \frac{\nu}{1+\nu} \right) \frac{\sigma}{2G} \delta_{ij} \quad (4)$$

where

$$\sigma = \sigma_{11} + \sigma_{22} + \sigma_{33} \quad (5)$$

and

$$\delta_{ij} = 1 \text{ when } i = j$$

$$\delta_{ij} = 0 \text{ when } i \neq j$$

The general equation (4) may be rewritten as follows in terms of the right side of equation (3) to include the inelastic strain

$$\epsilon_{ij} - \epsilon''_{ij} = \frac{\sigma_{ij}}{2G} - \frac{\nu}{(1 + \nu)} \frac{\sigma}{2G} \delta_{ij} \quad (6)$$

Equations (3) and (4) are the constitutive equations whereby a thermoelastic problem can be formulated as

$$\epsilon_{ij} - \epsilon''_{ij_T} = \frac{\sigma_{ij}}{2G} - \frac{\nu}{(1 + \nu)} \frac{\sigma}{2G} \delta_{ij} \quad (7)$$

Similarly, a creep problem can be formulated as

$$\epsilon_{ij} - \epsilon''_{ij_C} = \frac{\sigma_{ij}}{2G} - \frac{\nu}{(1 + \nu)} \frac{\sigma}{2G} \delta_{ij} \quad (8)$$

The same rationale can be extended to a collective problem of creep and temperature:

$$\epsilon_{ij} - (\epsilon''_{ij_T} + \epsilon''_{ij_C}) = \frac{\sigma_{ij}}{2G} - \frac{\nu}{(1 + \nu)} \frac{\sigma}{2G} \delta_{ij} \quad (9)$$

Equations (7), (8), and (9) lead to several important observations. First, it is obvious that equations (7) and (8) have a computationally analogous form. This allows the conclusion that a creep problem can be computed as a thermoelastic problem by equating

$$\epsilon''_{ij_C} = \epsilon''_{ij_T} \quad (10)$$

For a thermoelastic problem, the inelastic strain is

$$\epsilon''_{ij_T} = \alpha T \quad (11)$$

Secondly, equation (9), which states the collective problem of temperature and creep effects, also provides the basis for the use of finite-element software for a combined thermoelastic and creep analysis in a piecewise linear manner. A piecewise linear analysis of a problem in which temperature and creep effects are present is approached by combining the creep strain  $\epsilon''_{ij_C}$  at a discrete time  $t_m$  with the temperature strains  $\epsilon''_{ij_T}$  so that a composite inelastic strain  $\epsilon''_{ij_{CT}}$  is created. This composite strain can be represented as

$$(\epsilon''_{ij_T} + \epsilon''_{ij_C})_{t_m} = (\epsilon''_{ij_{TC}})_{t_m} = (\alpha T)_{t_m} \quad (12)$$

Equation (12) can be used in conjunction with equations (7) to (9) to form isotropic elasticity relationships.

## Finite-Element Methodology

Since it was established in the previous section that a creep problem can be computed as a temperature problem, the procedure using finite-element structural computer programs is discussed. The basic problem of computing residual stresses due to creep in a built-up structure is readily suitable to a piecewise linear analysis. This is particularly true if the new, very fast computers are used. A basic flow diagram presented in figure 1 illustrates the approach.

Since the necessity for studying creep in an airframe application is the presence of elevated temperature, it must be assumed that a thermostructural analysis is required. Therefore, the basic sequence begins by making geometric, material, force, and temperature inputs from which element stresses are computed. The element stresses are then input to an appropriate creep law and it is determined which of the elements are creeping. If no elements have temperature and stress combinations that result in creep, then the stresses are static and there is no creep problem. However, if one or more elements are creeping, then an amount of creep strain is computed from the creep law for each element, based on the particular stress and temperature situation for that element. This amount of creep deduced from the creep law is also based on some predetermined time interval of suitable convergence.

The amount of creep strain occurring in each creeping element must then be converted to an equivalent thermal strain  $\alpha T$ . This is most easily accomplished by adjusting the coefficient of thermal expansion  $\alpha$  for each creeping element. Then, in the case of a transient problem, different temperatures and forces are resubmitted and a new set of element stresses is computed for comparison with the creep law. Additional creep strains are compiled and reduced to  $\alpha T$  inputs so that the cycle can be repeated for more time increments. Using this process, the operating stresses and changes in operating stresses with time are identified.

The total residual stress caused by creep at the end of  $m$  time cycles is computed from cumulative creep strains of the individual elements. This is a single computation with the cumulative individual creep strains represented by the quantity  $\alpha T$ . The first step is applying a uniform temperature to the structure, then altering the coefficient of thermal expansion of each of the creeping elements so that the alteration of the quantity  $\alpha T$  equals the cumulative creep in that element. If a problem entails a large number of creeping elements, then a significant amount of extra labor is required to produce additional element property and material cards to describe the problem. Problems in which there are very few creeping elements and the elements are discrete (not connected to any other creeping elements) may be approached by altering the temperatures at the boundary of the element. This is discussed in some detail in the next section.

## Applications

Altering  $\alpha$  or  $T$ . - Consider the basic structural situation of figure 2, which consists of a system of five node points with four elements connecting these node points. The basic inelastic strain of the elements caused by temperature for this problem is

$$\epsilon''_{ij_{T_{kl}}} = \left( \frac{T_k + T_l}{2} \right) \alpha_{kl} \quad (13)$$

where  $k = 1, 2, 3, 4$ , and  $l = 5$ .

When the stresses resulting from a creep problem are computed using the thermo-elastic feature of any finite-element computer program, either the coefficient of thermal expansion  $\alpha$  or the temperature  $T$  must be altered an amount relative to the creep strain. If the problem is approached using the temperatures as the altered parameter, the strain in the individual elements must be changed by a factor,  $\eta_{kl}$ :

$$\epsilon_{ij}''_{T_{kl}} = \eta_{kl} \left( \frac{T_k + T_l}{2} \right) \alpha_{kl} = \left( \frac{T_k + T_l}{2} \right) \alpha'_{kl} \quad (14)$$

where  $k = 1, 2, 3, 4$ , and  $l = 5$ .

Unless the creeping elements are isolated, discrete, and not connected to any other creeping elements, a major computational dilemma exists. Consider the case where two of the elements of figure 2 (1-5 and 3-5) are creeping. If the problem is approached by altering the temperatures, then for element 1-5 the appropriate change in inelastic strain is made by altering the average temperature of the element by  $\eta_{kl}$  such that

$$T_{AVG} = \eta_{15} \left( \frac{T_1 + T_5}{2} \right) \quad (15)$$

for the element 3-5

$$T_{AVG} = \eta_{35} \left( \frac{T_3 + T_5}{2} \right) \quad (16)$$

These inputs result in temperatures of  $\eta_{15} T_1$ ,  $\eta_{35} T_3$ , and  $\eta_{15} \eta_{35} T_5$ , for node points 1, 3, and 5, respectively.

The overall input is obviously inconsistent at point 5 for the individual element requirements of equations (15) and (16). This result becomes more incorrect as additional creeping elements are connected to point 5. Therefore, a general rule can be stated: If creeping elements are not discrete and are connected to other creeping elements, the problem may not be approached by analogy as a thermoelastic problem by altering nodal temperatures. The solution to such a problem requires the alteration of the individual coefficients of thermal expansion of the individual elements.

To solve the figure 2 problem, do not alter inputs directly to the nodes, but alter inputs directly to the elements. Altering the material property cards, the approach for the figure 2 problem would be

$$\alpha'_{15} = \eta_{15} \alpha_{15} \quad (17)$$

and

$$\alpha'_{35} = \eta_{35} \alpha_{35} \quad (18)$$

Equations (17) and (18) allow inputs consistent with equations (15) and (16), regardless of how many elements within the system are creeping or how they are interconnected.

Uniformly inelastically strained element. — A frequent uncertainty concerns the relationship between free thermal expansion and stress. Consider a completely unrestrained elastic body with a uniform initial temperature. The body is then subjected to a uniform temperature change, resulting in thermal deformation. The situation is also possible for creep or plasticity. Because there is dimensional change — and strain is defined as a dimensional change — why is there no stress? The answer lies in the fact that the total strain and the inelastic strain ( $\alpha T$ ) are equivalent and there is no elastic strain, hence, no stress. This is demonstrated in equations (3) and (6) where the total strain is equal to the deformation and the inelastic strain is equal to  $\alpha T$ , which is also the same as the deformation, or

$$\epsilon_{ij} = \epsilon''_{ij} = \alpha T$$

therefore,

$$\epsilon'_{ij} = 0$$

and hence

$$\sigma_{ij} = 0$$

If the deformation is in any way restrained, then

$$\epsilon_{ij} \neq \epsilon''_{ij} = \alpha T$$

therefore,

$$\epsilon''_{ij} \neq 0$$

and hence

$$\sigma_{ij} \neq 0$$

The solutions of inelastic problems can be changed to those of elastic problems by a general analogy between body forces and inelastic strain gradients. The general analogy is that the strain distribution in a body subject to a given set of body and surface forces with inelastic strain is the same as the strain distribution in an identical body with no inelastic strain, but with an additional set of body and surface forces. References 6 and 7 provide the analogy background. The special application to the temperature problem is sometimes known as Duhamel's Analogy.

Creep law convergence. — Because the basic computational approach of figure 1 depends on piecewise linear sequencing, it is extremely important to evaluate the convergence of the computational time increment. A creep experiment described in reference 1 and a creep law obtained for the titanium alloy Ti-6Al-4V reference 2 are used to examine a convergence. The following creep law was examined for stresses and temperatures typical of a future airframe:

$$\ln \epsilon''_{ijC} = -24.09 + 22.54T + 0.000006 \sigma^2 + 0.905 \ln \sigma + 0.433 \ln t \quad (19)$$



The creep law was examined for convergence for several computational time increments ranging from 0.5 to 6.0 hr. The results are presented in figure 3. A convergence to within 5 percent of the asymptote can be achieved with time increments as small as 1 hour. This result will vary with other materials, hence, the result must be interpreted to see if the material examined displays the classic primary and secondary creep behavior. Figure 4 shows creep strain as a function of time for the Ti-6Al-4V material (ref. 8). Definite primary and secondary effects are shown to develop in less than 20 hr. Twenty hours of incremental analysis (20 to 40 iterations) provided a relevant answer, pointing out the significance of the creep residues for most vehicles contemplated at this time. This indicator approach appears feasible because the large numbers occur early in the problem (primary creep), making the seriousness of the creep evident.

Determining a valid creep law may well be the most formidable challenge in terms of predicting creep residues in high-speed airframes. It has been established (ref. 9) that there is a great difference between steady-state creep laws and cyclic creep laws. There is also evidence (ref. 3) that besides temperatures, time, and stress the material thickness must also be considered in defining a creep law. Therefore, when approaching creep effects in airframes, the difficulty of establishing the important peripheral item of a valid creep law must not be overlooked.

#### CONCLUDING REMARKS

When a new airframe is being developed, there is a strong tendency to rely on proven, demonstrated computational approaches to design and analysis. This tendency has been a strong motivator in the logic of the computational approach presented in this paper. The analogy between the inelastic strains caused by temperature and creep is presented in terms of isotropic elasticity. It shows how the theoretical aspects can be blended with existing finite-element computer programs to exact a piecewise linear solution. The creep effect can be determined by using the thermal stress computational approach, if appropriate alterations are made to the  $\alpha T$  of the individual elements. The overall transient solutions can be achieved by consecutive piecewise linear iterations. The total residue caused by creep can be obtained by accumulating creep residues for each iteration, then the total residues for each element are resubmitted as an equivalent input.

A typical creep law was tested for incremental time convergence. The results indicated that the approach was quite practical, with a valid indicator of the extent of creep present after about 20 hr of incremental time. The general analogy between body forces and inelastic strain gradients was discussed with respect to how an inelastic problem can be worked as an elastic problem.

## REFERENCES

1. Jenkins, Jerald M.; and Montoya, Christopher A.: Experimental Creep Data for a Built-Up Aluminum/Titanium Structure Subjected to Heating and Loading. NASA TM-84906, 1983.
2. Jenkins, Jerald M.: Effect of Creep in Titanium Alloy Ti-6Al-4V at Elevated Temperature on Aircraft Design and Flight Test. NASA TM-86033, 1984.
3. Dixon, S.C.; Tenney, Darrel R.; Rummler, Donald R.; Wieting, Allan R.; and Bader, R.M.: Structures and Materials Technology Issues for Reusable Launch Vehicles. NASA TM-87626, 1985.
4. Davis, George W.: Evaluation of Structural Design Concepts for High Speed Aircraft. NASA CR-3931, 1985.
5. Wang, Chi-Teh: Applied Elasticity. McGraw-Hill Book Co., Inc., 1953.
6. Duhamel, J.-M.-C.: "Second mémoire sur les phénomènes thermomécaniques." J. École Royale Polytech., vol. 15, no. 55, (Paris), 1837.
7. Duhamel, J.-M.-C.: "Mémoire sur le calcul des actions moléculaires développées, par les changements de température dans les corps solides." Mémoires présentés par divers savants, vol. 5, (Paris) 1838.
8. Aerospace Structural Metals Handbook. AFML-TR-68-115, vol. 4. Air Force Materials Lab., Wright-Patterson AFB, 1978.
9. Davis, John W.; and Cramer, Bruce A.: Prediction and Verification of Creep Behavior in Metallic Materials and Components for the Space Shuttle Thermal Protection System. NASA CR-2685, 1976.

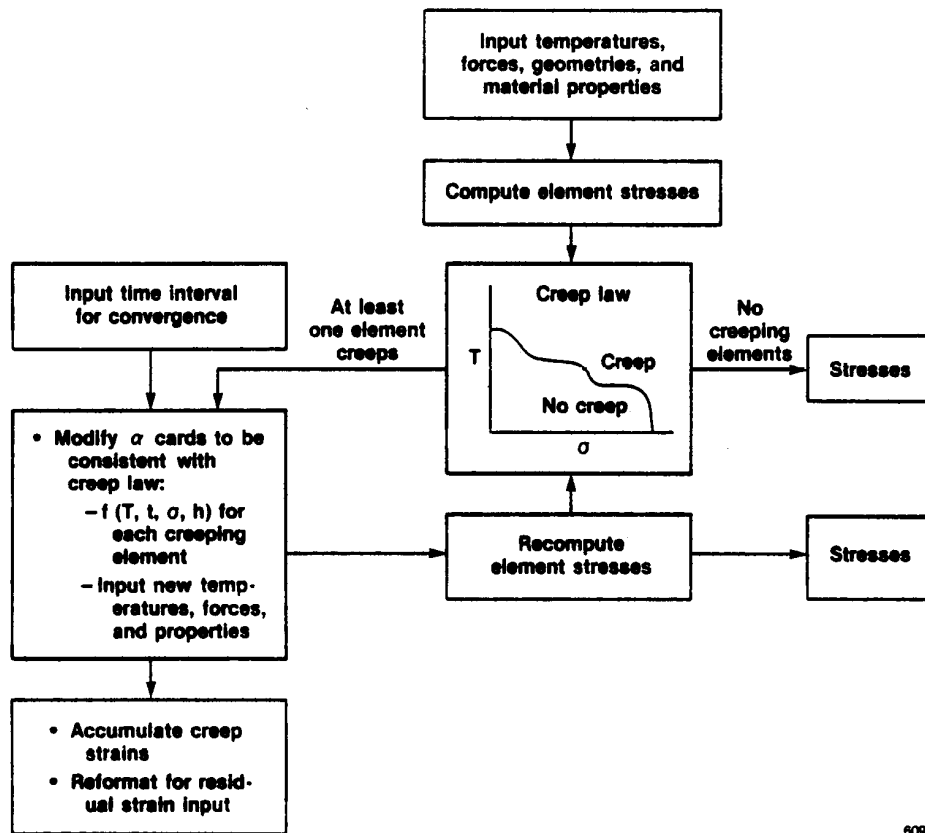


Figure 1. Computational flow diagram.

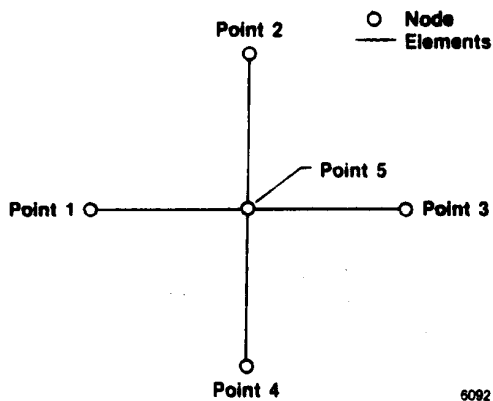


Figure 2. Structural arrangement of five node points and four elements.

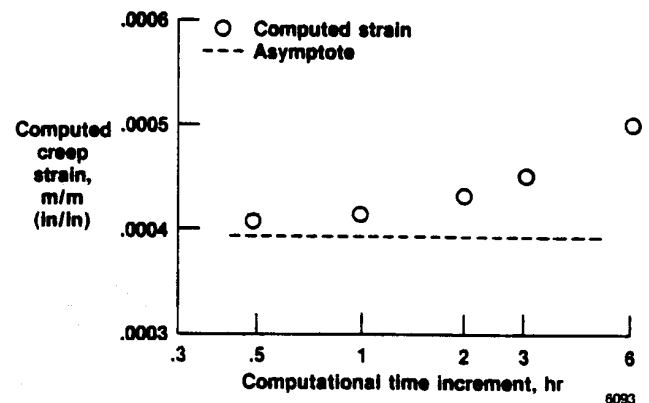


Figure 3. Creep law convergence for several time increments.

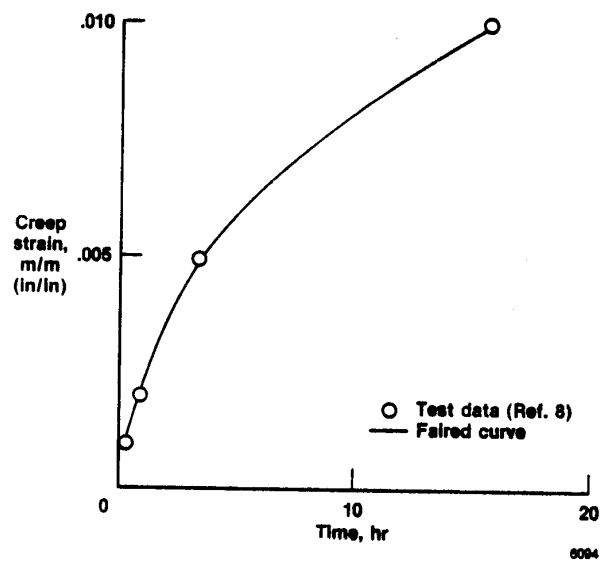


Figure 4. Typical creep curve for Ti-6Al-4V.

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